Probability (in Quantum Mechanics)

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To begin,
we need a general theory of probability.

Most general & inclusive:

Subjective Bayesianism

in which anything not known with certainty
can be assigned a probability
restricted only by
consistency with the axioms.

Probabilities are assigned to *statements* that must obey a logical calculus:

$S =$ a statement.

$\Omega =$ a statement known to be true.

$\emptyset =$ a statement known to be false.

$\overline{S} =$ a statement that is true iff S is false.

$S_1 \vee S_2 =$ a statement that is true iff either S_1 or S_2 is true.

$S_1 \wedge S_2 =$ a statement that is true iff both S_1 and S_2 are true.

$S_2|S_1 =$ a statement iff S_1 is true, true iff S_2 is true.

Probability assignments
must obey the axioms:

Axiom 1. $P(S)$ is a nonnegative real number.

Axiom 2. $P(S) = 1$ iff S is known to be true.

Axiom 3. If $S_1 \wedge S_2 = \emptyset$, then $P(S_1 \vee S_2) = P(S_1) + P(S_2)$.

Axiom 4. $P(S_2|S_1) = P(S_1 \wedge S_2)/P(S_1)$.

Usually in math,
the axioms are considered to be
a sufficient definition
of whatever obeys them.

Q: What statements are allowed?

A: Any assertion of a putative fact.

"When this coin is flipped it will come up heads."

"It rained here yesterday."

"The value of Newton's constant is between 6.6 and $6.7 \times 10^{-11}\,\mathrm{m^3/kg\,s^2}$."

"Beethoven was a better composer than Berlioz."

"The probability that it rained here yesterday is between 0.66 and 0.67."

We are not claiming that [a probability of a probability] is a real "probability" in the sense that we have been using that term; it is only a number which is to obey the mathematical rules of probability theory.

E. T. Jaynes
Probability Theory: the Logic of Science
(Cambridge, 2003)

If you don’t like
probabilities of probabilities,
we can invoke the
di Finetti representation theorem
(classical or quantum)
which says that they’re OK
(at the cost of constructing
an artificial sample space)

Bayes' theorem

(follows immediately from Axiom 4)

Data → $H|D$; Hypothesis → $D|H$

$$\underbrace{P(H|D)}_{\text{"Posterior"}} = \underbrace{P(D|H)}_{\text{"Likelihood"}}\,\underbrace{P(H)}_{\text{"Prior"}}\,/P(D)$$

A key point:

This or that “principle”
is often just
somebody’s choice of prior.

Subjective Bayesians
always have the option of
agreeing or disagreeing
with that choice.

Example: Boltzmann brains

Subjective Bayesian working assumption:

P=1 *P*=0

"Principle of Indifference": $P=1/N$

These are just different
xerographic distributions.

No need to make a definitive choice!

e.g.,

P(SBwa)=0.95

P(PoI)=0.05

Why should we doubt the
"Principle of Indifference"?

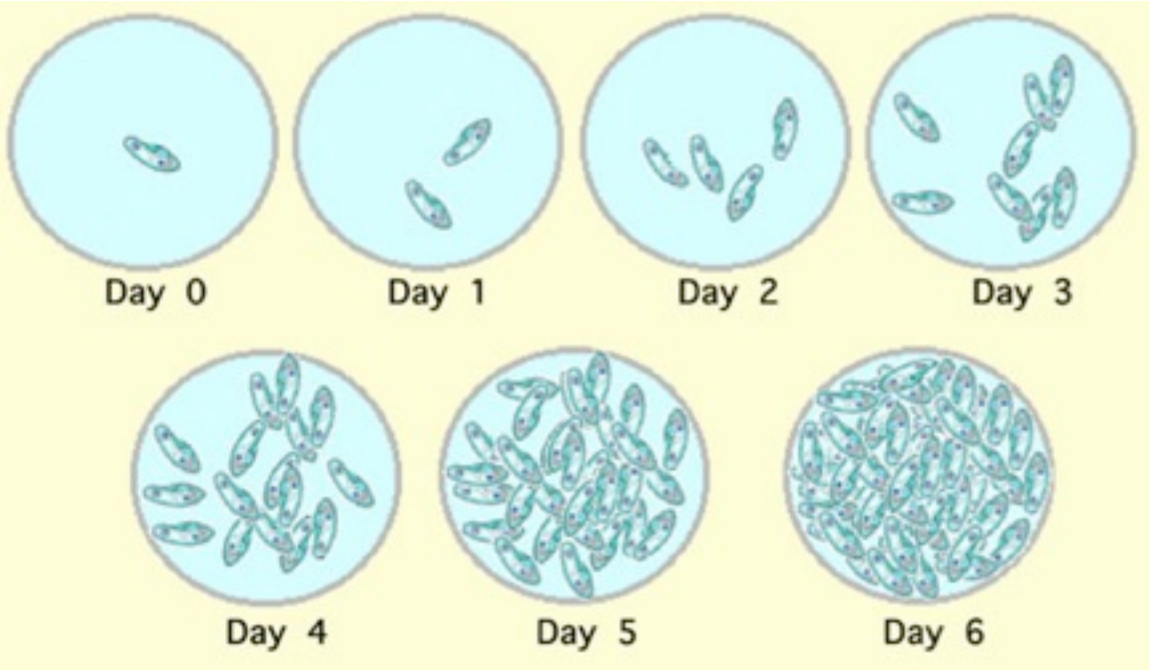
Why should we doubt the "Principle of Indifference"?

Because it's known to be false!

Example: Are humans typical animals on Earth?

No!

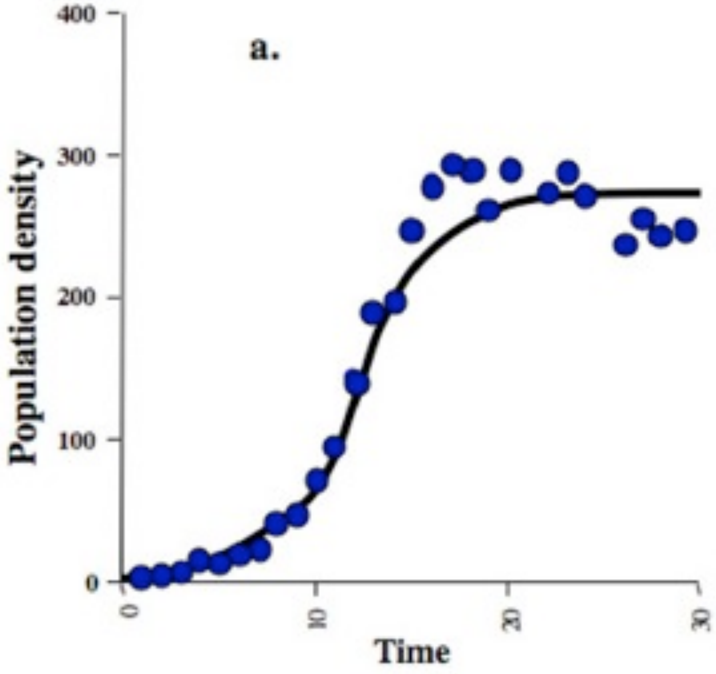
Downsizing Doomsday:



Under specified conditions,
will a bacteria colony grow exponentially?

Pol: No

Expt: Yes



Nature Education Knowledge Project

Morals:

Subjective Bayesian framework
(with probabilities of probabilities allowed)
can accommodate all forms of reasoning
in actual use by human beings.

Choices of priors
(= “principles”)
are *assumptions*
that should be subjected
to scientific scrutiny and doubt.

Probability in QM:

ontic

or

epistemic?

Epistemic: need QM generalization
of SB framework (hard!)

Ontic: Born's Rule = physical law;
SB probabilities must conform

Decoherent histories

Exact (medium) decoherence
needed to define ontic probabilities:

$$C_\alpha = P^n_{\alpha_n}(t_n) \cdots P^1_{\alpha_1}(t_1)$$

$$\langle \Psi | C^\dagger_\beta C_\alpha | \Psi \rangle = 0, \quad \alpha \neq \beta$$

$$p(\alpha) \equiv ||C_\alpha|\Psi\rangle||^2 = \langle \Psi | C^\dagger_\alpha C_\alpha | \Psi \rangle$$

But what if decoherence is not exact?

Carroll & Sebens, Vaidman, ... :

State is ontic

Probability arises from
"branch location uncertainty"

$$|\Psi\rangle = |O\rangle|\uparrow\rangle|\omega_1\rangle + \sqrt{2}|O\rangle|\downarrow\rangle|\omega_2\rangle$$

$$= |O\rangle|\uparrow\rangle|\widehat{\omega}_1\rangle + |O\rangle|\downarrow\rangle|\widehat{\omega}_2\rangle + |O\rangle|\downarrow\rangle|\widehat{\omega}_3\rangle$$

Apply envariance Zurek

Apply PoI

Ontological assumption:

$$|\Psi\rangle = |O\rangle|\uparrow\rangle|\widehat{\omega}_1\rangle + |O\rangle|\downarrow\rangle|\widehat{\omega}_2\rangle + |O\rangle|\downarrow\rangle|\widehat{\omega}_3\rangle$$

implies 3 copies of *O*:

1 experiencing ↑

2 experiencing ↓

We can then apply PoI

or

pick some other xerographic distribution!

If this ontological assumption is true,
then it should hold to a good approximation
for slightly non-orthogonal ω's

Conclusions:

1. *Subjective Bayesianism*,
   with probabilities of probabilities allowed,
   and no rigid choice of priors,
   is the most general scientific framework.

2. Quantum probabilities must fit into it.

3. Ontic states and branch uncertainty (Carroll & Sebens) alleviates some puzzles of ontic (Born's Rule) probability.

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