NEGligible momentum transfers and anomalous decoherence: Dark matter detection with nanomechanical resonators?

C. Jess Riedel
with Itay Yavin
Perimeter Institute

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NEGLIGIBLE MOMENTUM TRANSFERS AND ANOMALOUS DECOHERENCE: DARK MATTER DETECTION WITH NANOMECHANICAL RESONATORS?

Betteridge's law of headlines:

“Any headline paper title that ends in a question mark can be answered by the word ‘no’.”

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Slides/video/blog at jessriedel.com
NEG LIGIBLE MOMENTUM TRANS FERS AND ANOMALOUS DECOHERENCE:
DARK MATTER DETECTION WITH NANOMECHANICAL RESONATORS?

Nanomechanical resonators unlikely to compete with matter interferometer until coherence is actually nanoscale:
Superposition breadth ~ O(1 nm)

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Orientation

Particle physics

Large superposition experiments
(optomechanics, nanomechanics, matter interferometers)

Quantum foundations
(decoherence, consistent histories)
Outline

- Initial motivation: bowling balls and ping-pong balls
- New SQL for detecting diffusion/decoherence
- Ideal searches for decoherence-based methods:
  - Tiny momentum transfers
  - Soft, not weak, interactions
- Candidate: Dark matter scattering through a heavy-photon mediator
  - Sketch search potential for matter interferometers and nanomechanical resonators
- Intuition and issues with sensitivity of nanomechanical systems
  - Need nanometer (not femtometer) coherence length scales
Suppose everything in the universe—including us—were made of bowling balls.

Now suppose we were surrounded by a sea of slow-moving ping-pong balls.

Could we ever tell?

Is it possible to influence without being influenced?
What are the limits on experimental physicists for identifying “sectors” of the universe to which we are weakly coupled?

- Dark matter
- Supersymmetry
- New neutrinos
- Mirror matter
- Fifth forces
- ...

Bowling balls and ping-pong balls
**“Force” Standard Quantum Limit**

- Suppose we need to measure a weak force $F$ during a short time period $T$ acting on a probe $M$
  - For example: gravitational waves, for which the time-averaged force is zero

- Suppose further that we are restricted to position (or position-like) measurements
  - Make initial position measurement
  - Wait time $T$
  - Make final position measurement

\[
\Delta x_F = \frac{T^2 F}{2M}
\]
For sufficiently weak forces, the wavepacket is simply not displaced enough to be detectable.

Narrowing the initial wavepacket does not help past a certain point:

\[ \frac{\Delta x}{\sigma_x} = \frac{\sqrt{\frac{\hbar T}{M}}} \ll \sigma_x \]

- Smaller initial width causes faster spreading during the time interval.

For optimal width, there is a smallest measurable displacement.

- The “standard quantum limit” (SQL):

\[ \Delta x_{\text{SQL}} = \sqrt{\frac{\hbar T}{M}} \]
Force Standard Quantum Limit

- Smallest measurable displacement implies weakest measurable force

\[ \Delta x_F = \frac{T^2 F}{2M}, \Delta x_{SQL} = \sqrt{\frac{\hbar T}{M}} \]

- This is based on a given probe mass \( M \) and a given time interval \( T \)

- Crucially, this assumes a particular preparation and measurement
Beating the force SQL

- Alternative: produce superposition of widely separated wavepackets in an interferometer
- The weak force is associated with a potential energy difference $U_{\text{eff}}$ between arms
- The potential energy difference produces a measurable phase shift

$$U_{\text{eff}} = -LF$$

$$\phi = U_{\text{eff}} T / \hbar$$
Beating the force SQL

- In principle can measure *arbitrarily* weak forces for sufficiently wide wavepacket separation $L$
- This is the origin of the extreme sensitivity to weak forces in atom interferometry
  - See Atomic Gravitational wave Interferometric Sensor satellite
  - Advanced LIGO uses intermediate techniques (squeezing)

\[ U_{\text{eff}} = -LF \]

\[ \phi = U_{\text{eff}} T / \hbar \]
Gravitational waves are weak \textit{classical} forces

A classical influence on a quantum probe can always be modeled as a unitary transformation

But Brownian baths and other sources of diffusion and decoherence cannot be modeled as unitary

- Rather, the sources may become entangled with the probe
We can prove an analogous limit for detecting collisional decoherence as for detecting weak forces.

The prototypical source of collisional decoherence is quantum Brownian motion (QBM).

In the appropriate special case, the idealized probe dynamics are

\[ \partial_t \rho = -i[\hat{H}, \rho] - \frac{D}{2} [\hat{x}, [\hat{x}, \rho]] \]

\[ \hat{H} = \frac{\hat{p}^2}{2m} - F \hat{x} \]
Force versus diffusion SQL

- $F$ parameterizes the strength of the force
- $D$ parameterizes the strength of diffusion and decoherence
  - $D$ induces random walk (diffusion) in momentum space
  - For terms sufficiently far off $\rho$’s main diagonal that $H$ can be ignored, we get pure decoherence

$$\partial_t \rho = -i[\hat{H}, \rho] - \frac{D}{2} [\hat{x}, [\hat{x}, \rho]]$$

$$\rho_t(x, x') \approx \rho_0(x, x') e^{-Dt(x-x')^2} \quad \text{for large } x - x'$$
Along with $F$, the coefficient $D$ is one of a handful of variables which parameterize a class of Markovian dynamics that

- generalize the Harmonic oscillator to open system and
- are uniquely preferred by symplectic symmetry

“Symplectic invariant” quantum Brownian motion

- Very satisfying topic if you want to get better intuition for the Wigner function and fully-general linear evolution in phase space
- Makes decoherence-diffusion connection transparent
- See PRA, appendix to arXiv:1507.04083, and citations therein
Force versus diffusion SQL

- **Force SQL:**
  \[ F_{\text{SQL}} = 2\sqrt{\frac{\hbar m}{T^3}} \]

  - Cannot detect forces
    \[ F \lesssim F_{\text{SQL}} \]
  without non-classical states and measurements

- **Diffusion SQL:**
  \[ D_{\text{SQL}} = \frac{9\hbar m}{8T^2} \]

  - Cannot detect diffusion
    \[ D \lesssim D_{\text{SQL}} \]
  without non-classical states and measurements
SQLs in phase space
Force versus diffusion SQL

- Just like for the traditional force SQL, diffusion SQL can be beaten by non-classical preparations like cat states and squeezed states.

- Cats in “wavepacket basis”:

\[
\begin{pmatrix}
\rho_{11} & \rho_{12} \\
\rho_{21} & \rho_{22}
\end{pmatrix}
\rightarrow
\begin{pmatrix}
\rho_{11} & e^{s+i\theta} \rho_{12} \\
e^{-s-i\theta} \rho_{21} & \rho_{22}
\end{pmatrix}
\]

\[
\theta = \frac{FLT}{\hbar}
\]

\[
s = \frac{DL^2T}{\hbar}
\]

Decoherence (diffusion)

Phase (force)
Classical undetectability

- These can detect *classically undetectable* phenomena
  - In $\hbar \to 0$ limit, wavepackets become points in phase space
  - Can simultaneously take $D$ and $F \to 0$, while phase shift and decoherence remain finite.

- Large superpositions are uniquely sensitive detectors

\[
F, D, \hbar \to 0, \quad s, \theta \text{ const.}
\]

\[
\theta = \frac{FLT}{\hbar}
\]

\[
s = DL^2T/\hbar
\]
Classical-to-quantum sensitivity

- More generally: Increased sensitivity to small momentum transfers comes as soon as you start cooling down a resonator mode.
- Once ground state is reached – the SQL – further sensitivity requires non-classical states:
  - Squeezed states
  - Cat states
• Sensitivity of cat states is proportional to $L^2$
• Reduces to SQL sensitivity as $L$ goes to zero

$$D_{\text{sense}} = \frac{\hbar^2}{L^2T}$$

$D_{\text{SQL}}$
Bowling-ball interferometry
Quantum superpositions can detect momentum transfer that can’t be detected any other way.

1: What sorts of new particles and forces fit the bill?
- *High* flux particles scattering *elastically* and *often* but transferring *tiny* amounts of momentum.

2: What’s best superposition probe (target)? Some considerations:
- Momentum sensitivity determined by superposition separation (coherence length), not recoil.
- Heavier targets reduce recoil but increase scattering cross-section.
- Important: “coherent elastic scattering”
Aside: Coherent elastic scattering

- Very general property of soft elastic scattering from targets composed of multiple \((N)\) charges
  - Not dependent on detail of particles or mediators
- When wavelength of incident particle is larger than target size, \(\lambda \gg R\), one gets \(\sigma \sim N^2\)
- When \(\tilde{\lambda} \ll \lambda \ll R\), there are complicated interference effects (constructive and destructive)
- Rule of thumb: boost is proportional to number of charges in “coherent scattering volume” (set by momentum transfer)
What type of superposition targets?

- We can superpose...
  - photons over many km, but they have no inertia
  - neutrons over macroscopic distances, but their inertia is tiny
- What are the heaviest objects we can superpose over a length scale that also contains them (guaranteeing crucial coherence enhancement)?
  - Implies superposed over distance larger than their physical size
- Answer: matter interferometers
  - Typical coherence lengths: ~5-200 nm
  - Typical object sizes: ~1-5 nm
  - Will compare to nanomechanical resonators
- So what should we try to detect?
How about dark matter?

- Many observations suggest new, non-baryonic form of gravitating matter
- Evidence comes from sub-galactic scales and above, e.g.
  - Galactic rotation curves
  - Bullet cluster
  - Large-scale structure
Basic dark matter

- All evidence is essentially gravitational
- Many, many competing ideas
- Candidate explanations must satisfy a wide range of experiments and observations stretching back decades
  - Many indirect, model-dependent restrictions
- Relatively few model-independent results
The dark matter halo

- But we have a **generic** local prediction: roughly spherical, virialized halo of dark matter enveloping the Milky Way
  - Isotropic in galactic rest frame
  - Maxwellian velocity distribution
  - Local density $\sim 0.4$ GeV/cm$^3$
  - Typical velocity $\sim 230$ km/s
- Assumed for limits set by underground detectors
- Based only on local, present-day observation
  - (no cosmology necessary)

Image source: European Southern Observatory (artist impression, duh)
Toward a bespoke theory of dark matter...

- Want scattering dominated by small momentum transfer \( \sim \hbar/(50 \text{ nm}) \)
- Otherwise, should be rarely interacting
- Recent idea from Itay Yavin: forward scattering off atoms through new massive scalar mediator ("heavy photon")
- Choose mediator Compton wavelength to be \( O(50 \text{ nm}) \)
  - Mediator mass: 1 - 100 eV
- Nonrelativistically, new force is bad at doing anything \textit{besides} transferring this much momentum to nuclei
  - \textbf{Warning}: model building – not well-motivated
Particle dark matter

- Model takes three key parameters:
  - Mediator-Nucleon coupling: $\alpha_{\text{B-L}} = g_M^2/4\pi$
  - Mediator mass: $m_{\text{B-L}}$ (with corresponding length scale $\lambda_{\text{B-L}}$)
  - Dark matter mass $m_{\text{DM}}$

- Further, this must be no more than an $O(10\%)$ component of DM
  - We imagine a dark sector that’s complicated, just like regular matter (bound states, multiple forces, etc.)
  - Would be just one part of a vast dark sea
  - Can construct arbitrarily complicated models to boost or suppress signal (e.g. large dark nuclei)
  - Sensitivity plots are just a guide
Interactions

- DM-nucleon interaction
- Nucleon-nucleon Yukawa potential

\[ V_M(r) = -\frac{g_M^2}{4\pi} \frac{1}{r} e^{-mr} \quad (5) \]

- Strongest constraints on \( \alpha = \frac{g_M^2}{4\pi} \) and \( \lambda_{B-L} = \frac{1}{m_{B-L}} \) come from Casimir-force and neutron-optics experiments
  - Nucleon-Nucleon rather than DM-Nucleon
Scatters in atm

Isotropized by atm

Neutron optics

EXCLUDED

$\alpha_{B-L}$

$\lambda_{B-L} (\text{m})$

$m_{B-L} (\text{eV}/c^2)$

$\lambda_{DM} = 1 \text{ MeV}, \alpha_{DM} = 1, \epsilon = 1$
Particle dark matter

- For flux reasons, we want heaviest DM mass that is consistent with
  - no visibility in rare-event detectors (e.g. neutrino or WIMP experiments)
  - unable to frequently induce electronic transitions
- Take $m_{DM} \sim 100$ keV – 3 MeV (and 1 MeV for plots)
- Energy deposition < eV even for rare head-on DM collision
  - Can say much more about why this is so hard to see otherwise
What’s the signal?

- For widely separated wavepackets in interferometers, DM would act as a source of decoherence.
Detection through decoherence

- Initial state: \[ |\mathcal{N}_L\mathcal{N}_H\rangle \mathcal{N}_R\rangle |D_{in}\rangle \]
- Final state: \[ |\mathcal{N}_L\rangle |D^{(L)}_{out}\rangle + |\mathcal{N}_R\rangle |D^{(R)}_{out}\rangle \]
- Measurement: \( \{ |\mathcal{N}_\pm\rangle = |\mathcal{N}_L\rangle \pm |\mathcal{N}_R\rangle \} \)

\[ \langle D^{(L)}_{out} | D^{(R)}_{out} \rangle \approx 0 \]
What’s the signal?

- For systems closer to the origin of phase space, DM would act purely as a diffusion/decoherence coefficient $D$
- This is slow for nanomechanical resonators
  - At relevant length scales, the characteristic diffusion rate $\sim D/x_{ZP}^2$ is always much smaller than resonator frequency $\omega_m$
- Appears as anomalous, weakly coupled, infinite-temperature bath
  - Characterized by phonon-injection rate $\sim D/x_{ZP}^2$
  - Similar to signals of speculative objective collapse models
Anomalous decoherence

- There are many possible sources of decoherence and noise
- Major challenge of nanomechanics is identifying and defeating one level of noise after another
- Anomalous decoherence does not imply dark matter
- However, the inverse statement is true: a cold (or quantum) resonator implies all sources of noise above some threshold have been eliminated
- This can establish robust dark matter exclusion limits
- But if we think anomalous decoherence might be due to dark matter, how could we be sure?
Establishing convincing evidence

- Try varying experimental parameters, e.g.
  - Size and shape of the resonator
  - Applied driving/cooling
  - Elemental composition of resonator
  - Isotopic composition of elements
  - (Analogous parameters exist for interferometers)

- General sources of decoherence will not have same dependence on these parameters
Establishing convincing evidence

- Try influencing expected dark matter flux
  - Shield experiment from dark matter (concrete, lead, underground)
  - Strength of dark matter wind will vary by order unity over day, and several percentage points over the year, due to Earth’s motion

- In general, the orientation of the resonator will give order-unity change to $D$
  - Resonators are naturally directional detectors!
  - Could unambiguously identify a signal possessing a fixed direction in the galaxy!
Let’s look at a few benchmark experiments to see the sort of sensitivity that’s possible for our proof-of-concept model of DM

- 3 matter interferometers
- 4 nanomechanical resonators
Interferometric benchmarks

- #1: Kapitza-Dirac-Talbot-Lau (KDTL) interferometer
  - Reached $>10^3$ amu (later $>10^4$)
  - Gerlich et al. *Natural Communications* 2, 263 (2011)
Interferometric benchmarks

- #2: Optical Time-domain Ionizing Matter-wave (OTIMA) Interferometer proposal
  - Forthcoming successor to KDTL interferometer with >10^6 amu

![Experimental layout and pulse sequence for an OTIMA interferometer.](image)

- A beam of nanoparticles is chopped into bunches, each of which is subsequently illuminated by the same three laser pulses (G₁, G₂, and G₃), separated by a variable but equal delay time of the order of the Talbot time T. The retro-reflection of the laser light at a common plane mirror generates a phase-stable standing wave. The laser wavelength is sufficiently short to ionize the particles with high probability at the antinodes after absorption of a single photon. The ions are removed by a homogeneous electric field. The remaining neutral particles fly into a time-of-flight mass spectrometer (TOF-MS), where they are photo-ionized, accelerated and detected by a multi-channel plate (MCP). This scheme allows one to post-select the clusters with a mass selectivity of better than 0.1% and to perform the experiment with a large number of different masses simultaneously. A quantum interference fringe pattern is observed in the count rate by varying the pulse separation T, provided the clusters are exposed to an additional constant acceleration a, e.g. due to gravitation or a constant electrical field gradient.

2.2. Near-field interference

Near-field TLIs are particularly well suited for exploring quantum wave mechanics with massive particles and short de Broglie wave lengths [12, 14, 16, 24, 25]. In our time-domain version, the laser pulses modify the x-component of the motional cluster state in the same way as the three gratings of a TLI 'in space'. The two concepts are related to each other by a change of the reference frame. Compared to a mechanical Talbot–Lau setup, the optical analogue adds substantial control due to the precisely defined delay between the pulses and the possibility of tuning the pulse strengths individually. This allows one to overcome source imperfections, in particular the longitudinal velocity spread in the beam, and to optimize the fringe visibility. In contrast to the spatial near-field interferometer, which is characterized by the Talbot distance $L_T = \frac{d^2}{\lambda}$,
Interferometric benchmarks

- #3: Optically trapped 120 nm silica nanoparticle on board the MAQRO satellite proposal
  - Nanoparticle suspended, cooled to COM motional ground state, then brought into superposition
  - Tech under development. See M4 mission proposal made to European Space Agency (arXiv:1503.02640)
$\lambda_{B-L} (\text{m})$

$m_{B-L} (\text{eV}/c^2)$

Scatters in atm

Isotropized by atm

$\alpha_{B-L}$

(\(m_{\text{DM}} = 1 \text{ MeV}, \ \alpha_{\text{DM}} = 1, \ \epsilon = 1\))
$\lambda_{B-L} = \frac{m_{B-L}}{c^2}$

Scatters in atm

Isotropized by atm

$(m_{DM} = 1 \text{ MeV}, \alpha_{DM} = 1, \epsilon = 1)$

- MAQRO interferometer
- OTIMA interferometer
- KDTL interferometer

$\alpha_{B-L}$

$m_{B-L}$ (eV/c$^2$)
Nanomechanical benchmarks

- #4: Si$_3$N$_4$ membrane-in-the-middle in Jack’s lab
Nanomechanical benchmarks

- #5: NIST/Boulder microwave resonator in ground state
Nanomechanical benchmarks

- #6: Vienna pulse-driven cantilever proposal
  - Single beam-split photon phase-space shifted to drive cantilever in weak-coupled but pulsed regime
Nanomechanical (?) benchmarks

- #7: Macquarie magnetically-levitated superconducting ring proposal
  - Very ambitious, very large superposition separation (~2 nm)
  - Johnsson et al arXiv:1412.6864
MAQRO interferometer
OTIMA interferometer
KDTL interferometer

\[ \lambda_{B-L} (\text{m}) \]

\[ m_{B-L} (\text{eV}/c^2) \]

Scatters in atm
Isotropized by atm

\( m_{DM} = 1 \text{ MeV}, \alpha_{DM} = 1, \epsilon = 1 \)
Macquarie levitated ring
Vienna pulsed cantilever
NIST/Boulder resonator
Yale cavity membrane
MAQRO interferometer
OTIMA interferometer
KDTL interferometer

\[ m_{B-L} (\text{eV}/c^2) \]

\[ \alpha_{B-L} \]

Scatters in atm
Isotropized by atm

\( (m_{DM} = 1 \text{ MeV}, \alpha_{DM} = 1, \epsilon = 1) \)
Interferometers vs resonators

- Where does the interferometer advantage over resonators come from?
- What’s the chief determinant of vastly different resonator sensitivity?
- Answer: Spatial coherence, i.e., superposition breadth
Interferometers vs resonators

- **Interferometers**
  - $10^4 - 10^{6.5}$ amu \((10^{10} \text{ in space})\)
  - 1-5 nm size
  - 50 – 200 nm separation

- **Typical resonators**
  - $10^{12} - 10^{16}$ amu
  - 1-100 nm thick
  - \((10 \text{ nm})^2 - (1 \text{ mm})^2\) wide
  - 0.01 – 10 fm separation
- Let’s do an extremely rough order-of-magnitude estimates
- Use OTIMA in Vienna and membrane-in-cavity from Jack’s lab as examples
- Ignore lots of import complications like
  - material composition
  - differing coherence enhancements for different momentum transfers
  - superposition lifetime and shape
\[ \sigma \sim N \times N \times N \times \frac{\Delta x}{d} \times \frac{\Delta x}{d} \times \frac{\lambda^2}{L^2} \]

\[ = N^2 \times \left( \frac{\Delta x^2}{d^2} \right) \times \left( \frac{\lambda^2}{L^2} \right) \]

\[ = (10^{6.5})^2 = 10^{13} \]
\[ \Delta x \sim 1 \text{ fm} \]

\[ L \sim 1 \text{ mm} \]

\[ d \sim 50 \text{ nm} \]

\[ \lambda \sim 50 \text{ nm} \]

\[ R \sim 3 \text{ nm} \]

$\sigma \sim N \times N$

\[ = (10^{6.5})^2 = 10^{13} \]

\[ \sigma \sim (N \times \Delta x/d) \times (N \times \Delta x/d \times \lambda^2/L^2) \]

\[ = N^2 \times (\Delta x^2/d^2) \times (\lambda^2/L^2) \]

\[ = (10^{16})^2 \times (10^{-8})^2 \times (10^{-4})^2 \]

\[ = 10^8 \]
Interferometers vs resonators

- When you throw in the additional complications, the discrepancy is even worse
  - $> 10^8$

- Worst penalty comes from short coherence length, which is squared
  - $(1 \text{ fm} / 50 \text{ nm})^2$
  - Can’t overcome this even when resonator is macroscopic!
Any hope?

- Need to pick up several orders of magnitude on the net where...
  - Resonator dimensions and superposition lifetime contribute linearly
  - Spatial coherence contributes quadratically

- If nanomechanics wants to be relevant as a detector of decoherence, the coherence needs to be truly *nanoscale*
  - I.e., superposition separation (or zero-point widths) not much smaller than a nanometer

- Pulsed lasers offer coherence wider than zero-point width...
  - ...but optical shot noise and thermal phonons on mechanical supports makes orders-of-magnitude improvements look daunting (to me)
Les Houches Brainstorm

- Here are a couple crazy and/or boring ideas for mesoscopic resonators with nanoscale coherence
Les Houches Brainstorm

- Mount mirror on superconductor and levitate?
Les Houches Brainstorm

- Detached mirrors, in free-fall or supported by radiation pressure?

Kasevich group, Stanford
The End

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