Branching of the Wavefunction of the Universe During Inflation

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Building on: EN, arXiv:1601.03734 (JCAP)
CJR, arXiv:1608.05377 (PRL)

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Goal: Find the branches of the wavefunction

We want

• mathematical account of measurements as a physical process

• to remove ambiguity about if/when measurements happen

We are not challenging

• fundamental indeterminism

• non-locality (failure of local realism)

• your interpretation quantum mechanics
Branches

At each level, branches must
• be mutually orthogonal
• sum to $|\psi\rangle$
Alternative justification: multipartite entanglement

Forget measurement

All claims will boil down to statements about a certain type of multipartite entanglement

Correlations in pure state between spacelike separated regions
Key observation: measurements make many records

Dynamical, irreversible process of entanglement:

\[ |\Psi\rangle = |\mathcal{E}\rangle \otimes |\text{lab}\rangle \otimes (|L\rangle + |R\rangle) \]

\[ \rightarrow |\mathcal{E}\rangle \otimes \left( |\text{lab}^{(L)}\rangle |L\rangle + |\text{lab}^{(R)}\rangle |R\rangle \right) \quad \text{...so that} \quad \langle \text{lab}^{(L)} | \text{lab}^{(R)} \rangle \approx 0 \]

\[ \rightarrow |\mathcal{E}^{(L)}\rangle |\text{lab}^{(L)}\rangle |L\rangle + |\mathcal{E}^{(R)}\rangle |\text{lab}^{(R)}\rangle |R\rangle \]

\[ |\Psi^{(L)}\rangle = \otimes_i |\mathcal{E}_i^{(L)}\rangle \quad \text{...so that} \quad \langle \mathcal{E}_i^{(L)} | \mathcal{E}_i^{(R)} \rangle \approx 0 \]

= MANY redundant records of the measured degrees of freedom
Key observation: measurements make many records

In words: the key feature of physical measurements is amplification.

Measuring devices take state of single system and imprint it on many systems.

Creation of a certain type of multipartite entanglement:

“GHZ-like” entanglement

N-qubit GHZ state: $|00 \cdots 0\rangle + |11 \cdots 1\rangle$
Production of records

Ubiquitous process in Nature — not just laboratories

1. Decoherence by bath with \textit{weak} self-interactions

\begin{itemize}
  \item Zeh, Joos, Zurek (“Quantum Darwinism”), the Horodeckis, Tegmark, etc.
\end{itemize}

2. Spontaneous symmetry breaking

\[
| ←← ⋯ ← \rangle \quad \rightarrow \quad | ↑↑ ⋯ ↑ \rangle + | ↓↓ ⋯ ↓ \rangle
\]
Key observation: measurements make many records

How to find this structure in large many-body systems?

What about multiple measurements?

GHZ states are very special

Need definition of records that is robust to real-world messiness, without assuming preferred

- observables
- basis
- systems-environment split
Defining records

Records are defined \textit{with respect to} a tensor product structure ("lattice"): \( H = \bigotimes_n H^{(n)} \)

A record is associated with a \textbf{region} (subset of the lattice)

\( F = \bigotimes_{n \in F} H^{(n)} \)
Defining records

A projector is **local** to a region when it acts trivially elsewhere:

\[ P^\mathcal{F} = P^\mathcal{F} \otimes I^{\bar{\mathcal{F}}} \]

\[ = \left[ \sum_{z \in P} |z\rangle^\mathcal{F}\langle z| \right] \otimes I^{\bar{\mathcal{F}}} \]

\[ \mathcal{H} = \mathcal{F} \otimes \bar{\mathcal{F}} \]

\[ P = P^2 = P^\dagger \]

\[ \langle z|z'\rangle = \delta_{zz'} \]

An observable is **local** when its eigenspaces correspond to local projectors:

\[ \Omega^\mathcal{F} = \left[ \sum_i \omega_i P_i^\mathcal{F} \right] \otimes I^{\bar{\mathcal{F}}} \]

\[ P_i P_j = \delta_{ij} P_i \]

\[ \omega_i \in \mathbb{R} \]
Defining records

Given a pure state and two disjoint regions, one local observable **records** another when their eigenvalues are fully correlated:

\[ P_i^F |\psi\rangle = P_i^{F'} |\psi\rangle \]

Equivalent: the reduced states at one region, **conditional** on different values at the other, are orthogonal

\[ \text{Tr} [\rho_{F',:i}^F \rho_{F',:j}^F] = 0, \quad i \neq j \]

\[ \rho_{F',:i}^F \equiv \text{Tr}_{\bar{F}} [P_i^F |\psi\rangle \langle \psi| P_i^{F'}] \]
Defining branches for one observable

Observers can infer the value of one observable by making a local measurement on another that records it.

Recording is a symmetric and transitive relation on local observables.

Each eigenvalue is associated with a branch:

$$|\psi_i\rangle \equiv P_i^F |\psi\rangle = P_i^{F'} |\psi\rangle = P_i^{F''} |\psi\rangle = \cdots$$

Branches are simultaneous eigenstates of all local observables.
Defining recorded observable and redundancy

A collection of local observables that record each other is a single recorded observable:

\[ \Omega \equiv \{ \Omega^\mathcal{F}, \Omega^\mathcal{F}', \Omega^\mathcal{F}'', \ldots \} \]

The number of records is the redundancy:

\[ R = |\Omega| \]
Now consider multiple recorded observables for many classical macroscopic outcomes

\[ \{ \Omega_1, \Omega_2, \Omega_3, \cdots \} = \{ \{ \Omega_1^F, \Omega_1^{F'} \}, \{ \Omega_2^G, \Omega_3^G \}, \cdots \}, \cdots \} \]
Multiple recorded observables

Now consider multiple recorded observables for many classical macroscopic outcomes
Multiple recorded observables

Now consider multiple recorded observables for many classical macroscopic outcomes
Multiple recorded observables

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Multiple recorded observables

Now consider multiple recorded observables for many classical macroscopic outcomes

[Diagram with labeled sets Ω₁, Ω₂, Ω₃, F', F'', F''', G', G'', I', I'', I]
Defining compatibility

A set of recorded observables are **compatible** if there is a joint branch decomposition

\[ |\psi\rangle = \sum_{i,j,k,\ldots} |\psi_{i,j,k,\ldots}\rangle \]

of simultaneous eigenstates

\[
\begin{align*}
\omega_i |\psi_{i,j,k,\ldots}\rangle &= \Omega_i^F |\psi_{i,j,k,\ldots}\rangle = \Omega_i^{F'} |\psi_{i,j,k,\ldots}\rangle = \cdots \\
\omega_j |\psi_{i,j,k,\ldots}\rangle &= \Omega_j^G |\psi_{i,j,k,\ldots}\rangle = \Omega_j^{G'} |\psi_{i,j,k,\ldots}\rangle = \cdots \\
\omega_k |\psi_{i,j,k,\ldots}\rangle &= \Omega_k^I |\psi_{i,j,k,\ldots}\rangle = \Omega_k^{I'} |\psi_{i,j,k,\ldots}\rangle = \cdots \\
&\vdots
\end{align*}
\]
Connection between records and branches

Key claim:

Set of recorded observables with “very large” redundancy is “very likely” to be compatible

Somewhat more precisely:

As $R's \to \infty$, regions must become pathological stringy to avoid compatibility

For details, see arXiv:1608.05377 [PRL, to appear]
Connection between records and branches

Some other claims:

- Very large redundancy is unlikely to be eliminated…until thermalization — irreversibility

- For very large redundancy, distinguishing between

\[ |\psi\rangle = \sum_i |\psi_i\rangle \quad \text{vs.} \quad \rho = \sum_i |\psi_i\rangle\langle\psi_i| \]

requires infeasibly non-local measurement — “collapse”

Large redundancy \(\leftrightarrow\) branches
Cosmology

How and when does branching happen in primordial cosmology?

\[ |\Psi(\varphi)\rangle \rightarrow \sum_{\varphi(x)} \]

- Pure state of the universe
- Different possible primordial fluctuations
- The realization we observe
Why cosmology?

Why consider perturbations during inflation?

- No observers or measuring devices in early universe — strains interpretation of quantum mechanics
- Realistic & popular “fundamental” system: relativistic QFT with gravity
- Redundant records as consequence of acceleration ($\ddot{a} > 0$)
- Tractable (many symmetries, large existing literature)
- Origin of all subsequent indeterminism / branching
Inflation review

Simplest model of slow-roll inflation

Only two fields: **metric** and **inflaton**

\[
S = \int d^4 x \sqrt{|g|} \left[ \frac{1}{2} R(g) + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right]
\]
Inflation review: classical solution

Flat, isotropic, spatially homogeneous classical solution:

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t) d\vec{x}^2 \]

Exponential expansion during slow roll

\[ a(t) \sim e^{Ht} \]

from shape of \( V(\phi) \)
Inflation review: quantum perturbations

Our quantum system: the scalar perturbation about the classical background

Gauge invariant quantity:

\[ \zeta(t, \vec{x}) \equiv \delta \left[ \ln \left( \frac{a(t, \vec{x})}{a_0(t)} \right) \right] - H \delta \left[ \frac{\phi(t, \vec{x})}{\dot{\phi}_0(t)} \right] \]

Classical solutions
Inflation review: quantum perturbations (linear)

Reduced Hamiltonian dynamics for linear theory ($\epsilon \to 0$):

$$H = \frac{1}{2} \int d^3 \mathbf{x} \left[ \frac{1}{a^3 \epsilon} \pi_x^2 + a \epsilon (\nabla \zeta_x)^2 \right]$$

In $k$-space, modes are decoupled in linear theory:

$$H = \frac{1}{2} \int d^3 \mathbf{k} \left[ \frac{1}{a^3 \epsilon} \pi_k^2 + a \epsilon k^2 \zeta_k^2 \right]$$

For each $k$, “horizon crossing” is characteristic timescale when

$$\lambda_{\text{phys}} = \lambda_H$$

$$\lambda_{\text{phys}} \equiv a(t)\hbar/k$$

$$\lambda_H \equiv c/H$$
Inflation review: quantum perturbations (linear)

Dilation symmetry under $(k, a) \rightarrow (\beta k, \beta a)$
Modes start in ground state
...become exponentially squeezed after horizon crossing:

\[ k \gg aH \quad \text{and} \quad k \ll aH \]

Wigner function
(phase-space representation of quantum state)

\[ \langle |\zeta_k|^2 \rangle \propto \left( \frac{k}{aH} \right)^2 \rightarrow 0 \]

Amplitude of fluctuations:
\[ \Delta^2 \zeta \equiv \langle \zeta^2 \rangle = \frac{H^2}{8\pi^2\epsilon} \sim 10^{-9} \]
Inflation review: quantum perturbations (interacting)

Even in simplest models, gravitational nonlinearities give interactions between \( k \)-modes

For minimal slow-roll, these are the first non-negligible interactions.

Generic: decoherence of large-wavelength modes by small-wavelength modes

EN, arXiv:1601.03734 [JCAP]

More complicated models can modify and introduce decoherence earlier, but this simple picture captures basic idea
Gravitational Nonlinearities

Consider (short)-wavelength $\zeta$ evolving within long-wavelength background $\zeta_L$

$$\mathcal{L}_0 + \mathcal{L}_{\text{int}} = \epsilon a^3 (1 + 3\zeta_L) \dot{\zeta}^2 - \epsilon a (1 + \zeta_L) (\partial \zeta)^2 + \ldots + \mathcal{O}(\epsilon^2)$$

Non-Gaussianity

### derivative interactions

A long-wavelength background $\zeta_L$ acts like a shift in $a(t)$

Squeezed-limit Bispectrum $\langle \zeta \zeta \zeta_L \rangle = 0$ (no shift in power spectrum)

But cross-bispectrum $\langle \pi \pi \zeta_L \rangle \neq 0$ (shift in time)

Conjugate momentum $\pi \equiv \delta \mathcal{L} / \delta \dot{\zeta}$

$$\propto \dot{\zeta} + 3\zeta \dot{\zeta} + \ldots$$

[Conjecture by Maldacena '03; Burrage+ '11; Creminelli+ '11, '12]
A tiny shift in time completely changes the squeezed state (for a shorter mode)

Precision = \frac{\Delta a}{a} \sim \frac{k}{aH} \ll 1
\sim \zeta_L

...the mode acquires a **record** of the long-wavelength background, with **increasing precision**
Field of Clocks

$\zeta_L(x)$

$\pi_S$
Branching structure for wavefunction of the universe

The shorter modes act as a recording environment for the longer modes.

\[
|\Psi\rangle = \left( \sum_{\varphi_S} \psi_S(\varphi_S) |\varphi_S\rangle \right) \left( \sum_{\varphi_M} \psi_M(\varphi_M) |\varphi_M\rangle \right) \left( \sum_{\varphi_L} \psi_L(\varphi_L) |\varphi_L\rangle \right)
\]

\[
\rightarrow \left( \sum_{\varphi_S} \psi_S(\varphi_S) |\varphi_S\rangle \right) \sum_{\varphi_M,\varphi_L} \psi_M(\varphi_M) |\varphi_M\rangle \cdot \psi_L(\varphi_L) |\varphi_L\rangle
\]

\[
\rightarrow \sum_{\varphi_S,\varphi_M,\varphi_L} \psi_S(\varphi_S) |\varphi_S\rangle \cdot \psi_M(\varphi_M) |\varphi_M\rangle \cdot \psi_L(\varphi_L) |\varphi_L\rangle
\]

Field configuration \( \varphi = \frac{\sqrt{2\epsilon}}{H} \zeta \)
Field of Clocks
Decoherence of Long-wavelength Mode

Integrate out part of the field: \( \rho(\zeta_L, \tilde{\zeta}_L) = \int \mathcal{D}\zeta_S \Psi[\zeta]|\zeta_L \Psi^*[\zeta]|\tilde{\zeta}_L \)

...to obtain mixed state for long-wavelength field

For a single long-wavelength mode \( \zeta_q \):

\[
\rho(\zeta_q, \tilde{\zeta}_q) \bigg|_a \propto \exp \left[-\Delta^2_{\zeta} \left(\frac{aH}{q}\right)^3 \cdot \frac{|\zeta_q - \tilde{\zeta}_q|^2}{2\langle|\tilde{\zeta}_q|^2\rangle} \right] \cdot \mathcal{O}(1)
\]

Density matrix becomes diagonal \( \frac{1}{3} \ln \Delta^{-2}_{\zeta} \approx 7 \) e-folds after horizon crossing \( (q = aH) \)

 Mostly from Hubble-scale environment modes (less precise clocks, but many more of them)

=system/environment hierarchy in scales
Measurement Basis (Pointer Basis)

Short mode cannot detect variation in $\zeta_L(\tau)$ as it freezes out,

$$\Delta \zeta_q \sim (q/aH)^2 \cdot \zeta_q^{(\infty)}$$

Precision of short-mode “clocks” does not grow fast enough to see this.

Similarly, decoherence does not occur in conjugate momentum basis

$$\rho(\pi(\zeta), \tilde{\pi}(\zeta)) \rightarrow \text{diagonal}$$

**Pointer basis $\approx$ field amplitude eigenstates** [Kiefer+ ’07, Burgess+ ‘14]
Wavefunction for long modes “collapses” to green pointer states following measurement by shorter modes

Or, exchanging momentum $\pi$

For physical velocity $\zeta$,
Redundant Records

Many clocks record the long-wavelength background

Redundancy (\# of records with precision $\Delta \zeta$)

$$\sim \frac{|\Delta \zeta_q|^2}{\sigma_\zeta^2} \sim \frac{|\Delta \zeta_q|^2}{\langle |\Delta \zeta_q|^2 \rangle} \Delta^2 \zeta \left( \frac{aH}{q} \right)^3$$
Lab measurements are special case of **amplification** (occurs naturally)

Amplification defined abstractly in terms of **redundant records**

(Strong correlations between many spacelike separated regions)

Sufficiently redundant records generate **branches**

Inflation provides clean model

**Accelerated expansion** turns scalar modes into sensitive clocks

**Gravity** couples modes, allowing the clocks to make a measurement

**Many modes** → redundant records & wavefunction branching
The End

EN, arXiv:1601.03734
CJR, arXiv:1608.05377

(Email jessriedel@gmail.com if you want a notification when the joint paper comes out.)