Classicality, Complexity, Observership, and Fault Tolerance

Charles H. Bennett
(joint work with Jess Riedel)

Quantum Foundations of
A Classical Universe

Sponsored by IBM and
John Templeton Foundation

13 August 2014
I will speak mostly of properties of quantum states, rather than dynamics.

Thermal disequilibrium (or fluctuations) sometimes give rise to **Classicality**, which is a necessary but insufficient condition for **Complexity**, which is a necessary but insufficient condition for **Observers**

Universal computation; algorithmic probability as a universal prior

Defining **Classicality** (via quantum Darwinism’s redundant correlations), **Complexity** (internal evidence in a classical state of a nontrivial computational history) **Observership** (internal evidence, in a complex state, of having practiced science)

Does the universe need to be fine tuned to produce complexity and observers? The universal prior gives a too easy answer of No. Should we introduce some physics, e.g. reversibility, noise?

Fault tolerance—stable memory, computation and self-organization despite hostile noise, i.e. without requiring fine tuning.
Sizeable regions where disequilibrium gives rise to Classicality and sometimes even Complexity and Observers.

Typical quantum state in a big Hilbert space is highly entangled, lacking classicality or any other interesting feature.

Boltzmann fluctuations, small and infrequent.
What does it mean for a state to be “classical?”

Information becomes classical by being replicated redundantly throughout the environment. “Quantum Darwinism”

In our out-of-equilibrium environment, scattered photons classicize events on the earth’s surface by broadcasting massively redundant replicas of them, in a preferred basis, into space.

System

Environment:

In 0/1 basis, system is correlated with each sub-environment. In other bases it is correlated only with the whole environment.
Defining complexity: We use a computerized version of the old idea of a monkey at a typewriter eventually typing the works of Shakespeare. Of course a modern monkey uses a computer instead of a typewriter.

A monkey randomly typing 0s and 1s into a universal binary computer has some chance of getting it to do any computation, produce any output.
The input/output graph of this or any other universal computer is a microcosm of all cause/effect relations that can be demonstrated by deductive reasoning or numerical simulation.
The Universal Semimeasure, or Universal Prior, or Algorithmic Probability, $P_U(x)$, is the probability that the monkey would cause the computer $U$ to embark on a terminating computation with the finite string $x$ as output.

Despite the obvious dependence on $U$, this deserves to be called universal because the ability of universal machines to simulate one another makes the definition machine-independent up to a multiplicative constant.

For any two universal machines $U$ and $V$, there exists a constant factor $f$ such that for all $x$, $P_U(x) / P_V(x)$ lies between $1/f$ and $f$.

(More on the universal prior later)
A simple cause can have a complicated effect, but not right away.
Self-organization, the spontaneous increase of complexity: A simple dynamics (a reversible deterministic cellular automaton) can produce a complicated effect from a simple cause.

Small irregularity (green) in initial pattern produces a complex deterministic “wake” spreading out behind it.
A sufficiently big piece of the wake (red) contains enough evidence to infer the whole history. A smaller pieces (blue) does not.
In the philosophy of science, the principle of Occam’s Razor directs us to favor the most economical set of assumptions able to explain a given body of observational data.

The most economical hypothesis is preferred, even if the deductive path connecting it to the phenomena it explains is long and complicated.
In a computerized version of Occam’s Razor, the hypotheses are replaced by alternative programs for a universal computer to compute a particular digital or digitized object $X$.

The shortest program is most plausible, so its run time measures the object’s logical depth, or plausible amount of computational work required to create the object.
To make logical depth more stable with respect small variations of the string $x$, and the universal machine $U$ a significance parameter $s$ is introduced. The $s$-significant depth of a string $x$, denoted $D_s(x)$, is defined as the least run time of any $s$-incompressible program to compute $x$:

$$D_s(x) = \min \{ T(p): U(p) = x \land |p| - |p^*| < s \}.$$ 

Here $p$ ranges over bit strings treated as self-delimiting programs for the universal computer $U$, with $|p|$ denoting the length of $p$ in bits, and $p^*$ denoting the minimal program for $p$, i.e. $p^* = \min \{ q: U(q) = p \}$.

This formalizes the notion that all hypotheses for producing $x$ in fewer than $d$ steps suffer from at least $s$ bits worth of ad-hoc assumptions. A near equivalent formulation is to say that $x$ has depth $d$ with significance $s$ iff less than $2^{-s}$ of the algorithmic probability of $x$ is contributed by programs running in time $<d$. 


A trivially orderly sequence like 111111… is logically shallow because it can be computed rapidly from a short description.

A typical random sequence, produced by coin tossing, is also logically shallow, because it essentially its own shortest description, and is rapidly computable from that. Depth thus differs from Kolmogorov complexity or algorithmic information, defined as the size of the shortest description, which is high for random sequences.
If a reversible local dynamics (e.g. the 1d system considered earlier) is allowed to run long enough in a closed system, comparable to the Poincaré recurrence time, the state becomes trivial and random.

Our world is complex because it is out of equilibrium.

After equilibration, typical time slice is shallow, with only local correlations.
At equilibrium, complexity still persists in 2-time correlations. Two time slices of the equilibrated system contain internal evidence of the intervening dynamics, even though each slice itself is shallow. The inhabitants of this world, being confined to one time slice, can’t see this complexity. (Also they’d be dead.)
In an equilibrium world with local interactions (e.g. a thermal ensemble under a local Hamiltonian) correlations are generically local, mediated through the present.

By contrast, in a non-equilibrium world, local dynamics can generically give rise to long range correlations, mediated not through the present but through a V-shaped path in space-time representing a common history.
The cellular automaton is a classical toy model, but quantum dynamics behaves similarly.

If the Earth were put in a large box and allowed to relax for a time comparable to its Poincaré recurrence time, its state would no longer be complex or even phenomenologically classical.

The radiation field in the box would no longer contain redundant optical replicas of details on the Earth’s surface. Rather the radiation field would be thermal, its photons having been absorbed and reemitted from the Earth many times. The entire state in the box would be a microcanonical superposition of near-degenerate energy eigenstates of the closed Earth+cavity system. Such states are typically highly entangled and contain only short-range correlations.
Having characterized classicality via quantum Darwinism, and complexity via logical depth, how do we define an observer?

Rather than focusing on consciousness, whatever that might be, we (Jess and I) take a rather different approach. Proceeding in the fashion of logical depth, we consider a string $x$ to contain an observer if it has internal evidence of having *practiced science*, that is of having made a more or less successful effort to understand and record a plausible explanation of its origin.

For example, let $x$ be a deep string, and $x^*$ be its minimal program. Then $x^*$ represents the most plausible explanation of the origin of $x$. Concatenating $x^*$ and $x$ produces the string $x^* x$ which is deep like $x$, but unlike $x$ also contains evidence (in the form of $x^*$) of having investigated and discovered its own most plausible computational origin. See us after class for more details.
Cosmologists worry about typicality, especially in connection with infinite universes, where it is hard to find a non-pathological prior distribution over “all possible universes”.

Cosmological models like eternal inflation resemble the rest of science in being based on evidence acquired from observation and experiment.

But could one instead try to define the set of “all possible universes” in a purely mathematical way, untainted by physics?

Yes—use the universal probability defined by the Monkey Tree, despite its being only semicomputable.

Having thus banished biology and physics from the prior, do we get a universe in which complexity and observers occur without fine tuning?

• Yes. But trivially so. However complexity and observership are defined, if the definition is computable, then it can be shown that a positive fraction of the monkey tree would have them.

Too Easy!

Maybe we should include some physics after all

Reversibility?
Superposition – quantum mechanics
Locality / field theories? (Lloyd and Dryer arxiv:1302.2850)

Another idea: insist that the monkey’s computer be fault tolerant—able to function reliably even in the presence of a certain amount of hostile noise. This would mean the computer would work without fine tuning of its transition probabilities.
How much fine tuning is required to get complexity, in the sense of logical depth, at thermal equilibrium?

Are thermal equilibrium states generically shallow? Yes.

- Gibbs phase rule: for generic parameter values, a locally interacting classical system, of finite spatial dimensionality and at finite temperature, undergoes nucleation and growth of a unique Gibbs state of lowest bulk free energy.

→ no long term memory
⇒ as $N, t \to \infty$, depth remains bounded

- Quantum exception, in 3 or more dimensions.
Classical **dissipative** systems can use anisotropic Toom-type voting to evade the Gibbs phase rule, storing information indefinitely and performing error-correcting computations despite hostile noise.

Toom’s NEC rule stable against generic symmetry-breaking field in 2d => Gacs-Reif fault tolerant cellular automaton in 3D
Phase Diagram of Classical Ising model in $d > 1$ dimension. Stores a classical bit reliably when $h=0$ and $T<T_c$.

Phase diagrams for local quantum models (Toric codes)*

- $d = 2$
  - $h = 0$
  - Stores a qubit at $T=0$. For $T>0$, stores a quantum-encoded classical bit, even when $h$ is nonzero, exception to Gibbs phase rule.

- $d = 3$
  - $h = 0$
  - Stores a qubit at $T=0$

- $d = 4$
  - $h = 0$
  - Stores a quantum-encoded qubit even at nonzero $T$ and $h$.

*Bravyi et al 0907.2807, Alicki et al 0811.0033…
Fault tolerant memory via dissipative processes (e.g. Toom)

Fault-tolerant Selforganization via Gacs’ 1d dissipative model with enforced hierarchical self-simulation

Classical or Quantum stable memory in non-dissipative models in low-dimensional (3 or 4) Toric codes.

Is there fault-tolerant selforganization at equilibrium in other manifolds, e.g. non-Euclidean, 4+1 dimensional?
Recent blog post on logical depth versus other complexity measures
http://dabacon.org/pontiff/?p=5912

C.H. Bennett "Logical Depth and Physical Complexity" in
The Universal Turing Machine– a Half-Century Survey, edited
Available at http://bit.ly/nh0bra

C.H. Bennett and G. Grinstein "On the Role of Dissipation in
Stabilizing Complex and Nonergodic Behavior in Locally

Charles H. Bennett, "How to Define Complexity in Physics, and
Why," in Complexity, Entropy, and the Physics of Information

C.H. Bennett “Quantum Information, the ambiguity of the past, and the complexity
of the present”, Perimeter Institute lecture 12 May 2011, URL:
http://pirsa.org/11050052/

A final question about information loss:

Blackbody radiation contains no information about the objects it illuminates. Does that mean it does not decohere them?
Looking inside a pottery kiln

by its own glow

by external light
Entangled Purification of Thermal input

$\Psi_\rho$

Different Purification out $\Psi'_\rho$
Sde Boker no restricted fonts
E75 Bose
Zurek
Newton Inst reasoning abt postselection
For KCIK
Plainness
1\textsuperscript{st} Q: Dynamics (Hamiltonian) probably needs to define subsyss but we will proceed as much as possible by statics: Diseq or Fluct $\rightarrow$ Classicality $\rightarrow$ Cxty $\rightarrow$ Observers \hspace{1mm} Venn diagram

Monkey Tree explain how it gives rise to universal prior

Class $\equiv$ QD, \hspace{1mm} cf Zurek clas via Redundant correlations from diseq or fluctuation $<$definable as a projector$>$

Cx $=$ logical depth $=$ internal evidence of long computation,

Intelligence/Observership $=$ internal evidence of mathematical or scientific activity

Universal prior and fine tuning $-$ too easy answers

Include some physics? Locality, reversibility, Lloyd’s Universal path integral.

Fine tuning in memory aka Fault Tolerance:

Gibbs phase rule

F.T. via diseq in memory (e.g. Toom)
FT Selforg, \hspace{1mm} via Gacs’ 1d enforced hierarchical self-simulation

C or Q stable memory w/o diseq in finite dim Toric codes
F.T. Selforg at eqb in d$>$1 dimension, cosmol manifolds