De Sitter Space Without Quantum Fluctuations

arXiv:1405.0298 (with Kim Boddy and Sean Carroll)

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Quantum Foundations of a Classical Universe
IBM Watson Research Center
August 12, 2014

What is a quantum fluctuation?

- Standard story: consider an observable $\mathcal{O}$ and a state $|\Psi\rangle$, not an eigenstate of $\mathcal{O}$. Then the variance $(\Delta\mathcal{O})^2_\Psi \equiv \langle\mathcal{O}^2\rangle_\Psi - \langle\mathcal{O}\rangle^2_\Psi \neq 0$, so repeated measurements of the state will have some scatter around $\langle\mathcal{O}\rangle$.
- This realization of nonzero variance by scatter in repeated measurements is what we mean by "quantum fluctuation."

What a quantum fluctuation is not

- Not a dynamical statement (present when we measure stationary states, $\frac{\partial}{\partial t} |\Psi\rangle \langle\Psi| = 0$ or $\dot{\rho} = 0$)
- Not a property of the state itself
- Instead, fluctuations are a property of the *interactions* between the system and a measurement apparatus.
- To talk about fluctuations as actual, physical events, need the whole machinery of decoherence.

Closed vs. Open Systems

- Fluctuations require a measurement apparatus. So it makes no sense to talk about fluctuations in a closed system.
- Intuition: a single isolated harmonic oscillator in its ground state “just sits there.” Nonzero variance in position, but no fluctuations. Same for any stationary state, e.g. thermal states.
- (Confusing issue: decoherent histories disagrees? Should discuss.)

Open Systems

- The dream: look at a reduced density matrix for a system. Check whether it fluctuates…
- …more precisely, check whether there are branches of the wave function on which fluctuations (Boltzmann brains, etc.) are present.
- Unfortunately, this is impossible...

Branching in the Decoherence Picture

- Decompose $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E$
- Write a general state $|\Psi\rangle = \sum_{ij} c_{ij} |s_i\rangle |e_j\rangle$
- Identify "pointer states" in $\mathcal{H}_E$, $\{|\hat{e}_j\rangle\}$
- Then $|\Psi\rangle$ has branched when we can write $|\Psi\rangle = \sum_j c_j |\hat{s}_j\rangle |\hat{e}_j\rangle$. This *defines* the system states $\{|\hat{s}_j\rangle\}$ which the system has branched into.

Caveats

- We can always decompose $|\Psi\rangle$ like this (Schmidt decomposition). Only physically relevant when the system states are correlated with the actual pointer states, though.
- We could have $\#\{|\hat{s}_j\rangle\} \gg \dim \mathcal{H}_S$ if the environment is large. E.g. the state of a single qubit could have three branches: $|\uparrow\rangle, |\rightarrow\rangle, |\leftarrow\rangle$. So the reduced density matrix ρ_S can't contain enough information to describe branching. Need to look at the entire wave function.

Extension to Multiple Systems?

- Could extend to a larger space with multiple systems + environments, $\mathcal{H} = \bigotimes_\alpha \mathcal{H}_S^{(\alpha)} \otimes \mathcal{H}_E^{(\alpha)}$.
- (Imagine e.g. α labels spatial position)
- $\rho^{(\alpha)} = \mathrm{Tr}_{\beta \neq \alpha} |\Psi\rangle \langle\Psi|$, $\quad \rho_S^{(\alpha)} = \mathrm{Tr}_{E_\alpha} \rho^{(\alpha)}$
- Impose partial-trace consistency? (c.f. Riedel, Zurek, and Zwolak on Quantum Darwinism, arXiv:1312.0331)
- Expect branching when

$$|\Psi\rangle = \sum_{i_a j_a} \tilde{\psi}_{i_a j_a} |i_a, j_a\rangle |\phi\rangle$$

Motivating Example

- Consider the de Sitter-invariant vacuum state for a massive scalar field (the "Hartle-Hawking vacuum.")
- Horizon-sized patch of dS has thermal density matrix, $T = H/2\pi, \; H = \sqrt{\Lambda/3}$.
- Does the Hartle-Hawking vacuum have branches on which fluctuations occur? To check, need to analyze the wave function.

The dS Wave Function

- In static coordinates, define creation and annihilation operators in the northern + southern hemispheres.

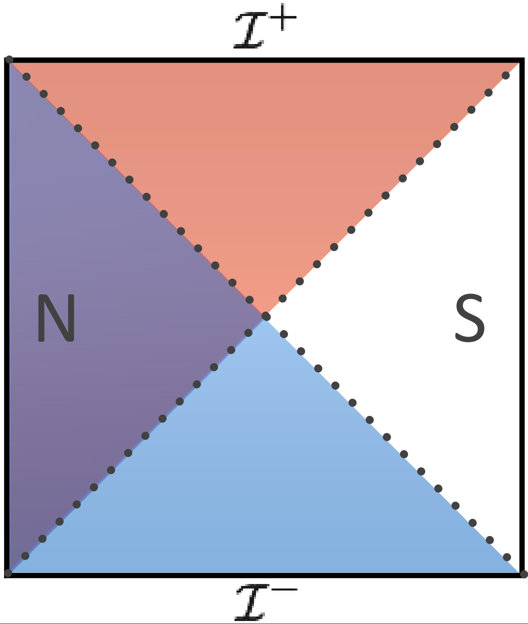
$$|\Omega\rangle = \prod_{\omega=0}^{\infty} \prod_{j=-\infty}^{\infty} \left(1 - e^{-2\pi\omega}\right)^{1/2} \exp\left[e^{-\pi\omega} \left(a^N_{\omega j}\right)^\dagger a^S_{\omega j}\right] |S\rangle \otimes |N\rangle$$

(Recall $a^S_{\omega j} = \left(a^S_{-\omega j}\right)^\dagger$)

- Define the static Hamiltonian: $\hat{H}_N = \int_0^\infty d\omega \sum_{j=-\infty}^{\infty} \left(a^N_{\omega j}\right)^\dagger a^N_{\omega j}\ \omega$

- Then the reduced density matrix is

$$\rho_N = \mathrm{Tr}_S |\Omega\rangle\langle\Omega| = \left[\prod_\omega \left(1 - e^{-2\pi\omega}\right)\right] e^{-\beta \hat{H}_N}$$



No Fluctuations in the dS Vacuum

- We see that:
- $|\Omega\rangle$ is static (it's the vacuum)
- ρ_N is static
- Modes in $\mathcal{H}_S$ are 1-1 correlated with modes in $\mathcal{H}_N$
- …so nothing is going on!
- In particular, there are no branches with localized excitations like Boltzmann brains.
- Why do we care?

The Cosmic No-Hair Theorem

- Intuition: any temporary structure in dS just dissipates over the horizon. So expect exponential decay of correlations.
- Wald 1983 (GR), Hollands 2010, Marolf and Morrison 2010 (QFT in dS), …
- Correlation functions of massive scalar fields decay exponentially. Decay constant is $\frac{3}{2}H$ for heavy fields.

The Vacuum is Inevitable

- So arbitrary perturbations around the Hartle-Hawking vacuum (e.g. us) will die down to the vacuum.
- Dissipative dynamics: violates unitarity?
- Not if $\dim \mathcal{H} = \infty$. This is true if we're describing regions outside a given causal patch.
- If horizon complementarity is valid, $\dim \mathcal{H} < \infty$, and can't actually reach the vacuum (have Poincaré recurrences).

dS without Boltzmann Brains

- No BBs in the vacuum, dS approaches the vacuum at late times → no BB problem for dS!
- Still have some finite expectation value for BB production in period before vacuum is reached (expect $\ll 1$). Provided more observers are produced "normally" (e.g. from structure formation), we can confidently conclude we're one of them and proceed to do science.

Applications beyond Stable dS

- Consider a more general potential.
- Slow-roll inflation
- Metastable vacua (e.g. inflationary/string landscape)

Easy Application: Slow-Roll Inflation

- How does inflation seed structure?

1. Inflaton dominates the universe, $w = -1$
2. Comoving horizon shrinks, superhorizon modes "freeze"
3. Reheating → entropy production (thermal bath of photons, etc.)
4. Comoving horizon expands, modes re-enter the horizon, contact environment, decohere.

This is unchanged in our picture.

The Stochastic Approximation

- The eternal inflation story since $\sim$1983 (Vilenkin, Linde, …):

1. During slow-roll, classical evolution decreases the inflaton field value by $\Delta\varphi$ every H^{-1}
2. Variance around $\langle\varphi\rangle$ is $\Delta^2 = \delta\varphi^2 = \frac{H^3}{4\pi^2}t$
3. Interpret Δ as a *physical* RMS fluctuation
4. Variance linear in time → random walk. Take steps of $|\delta\varphi| = \frac{H}{2\pi}$ every H^{-1}.
5. $|\delta\varphi| \gtrsim \mathcal{O}\left(\Delta\varphi/20\right)$ → one Hubble patch grows, eternal inflation.

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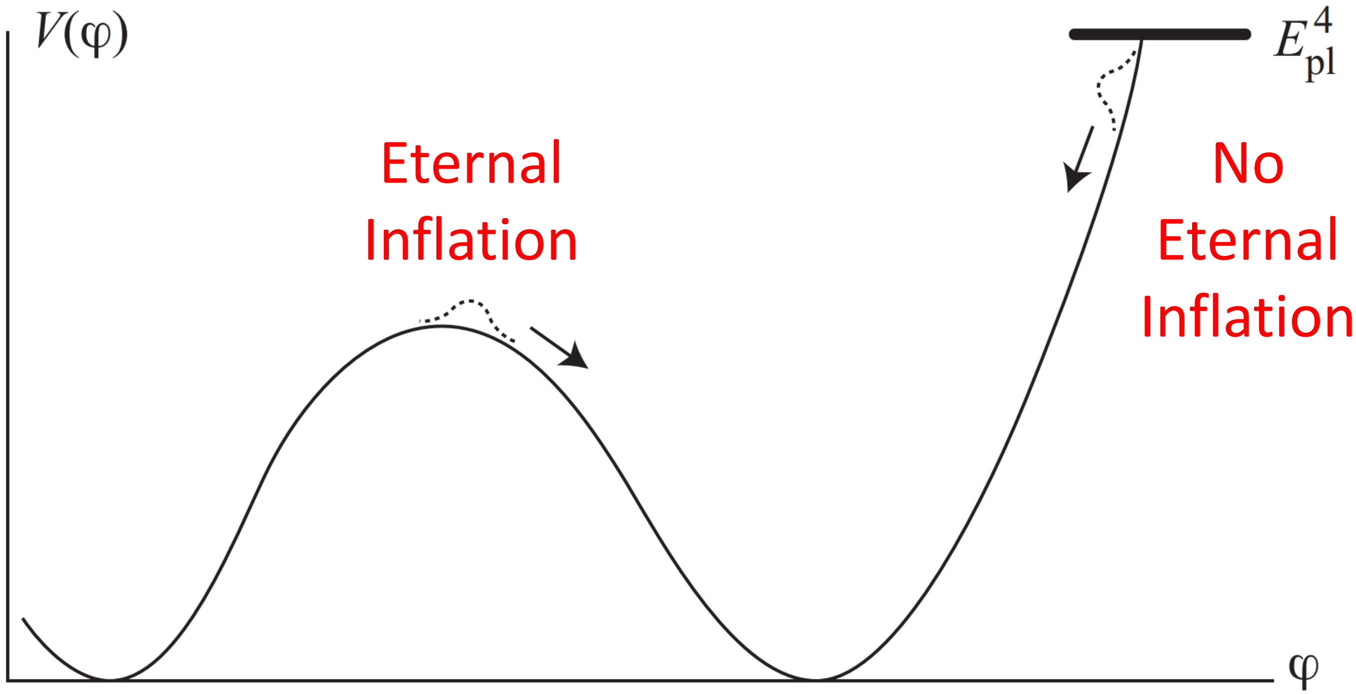
3. ~~Interpret Δ as a *physical* RMS fluctuation~~ **NO**
4. Variance linear in time → random walk. Take steps of $|\delta\varphi| = \frac{H}{2\pi}$ every H^{-1}.
5. $|\delta\varphi| \gtrsim \mathcal{O}(\Delta\varphi/20)$ → one Hubble patch grows, eternal inflation.

BAD

The Problem

- This is precisely what we're not allowed to do!
- In the absence of decoherence, can't interpret the variance of an observable as a "quantum fluctuation." Stop there. No need to use any of the results about de Sitter space…
- Recurring theme: all of the math is correct, but it doesn't answer the question we want answered: is inflation eternal?

No Eternal Inflation?



- If there's no decoherence during inflation, field will just roll down the potential until reheating. No eternal inflation from $\lambda\varphi^4$, $m^2\varphi^2$, or other monomial potentials. Need a local maximum or saddle point (e.g. hilltop inflation) to get something eternal.

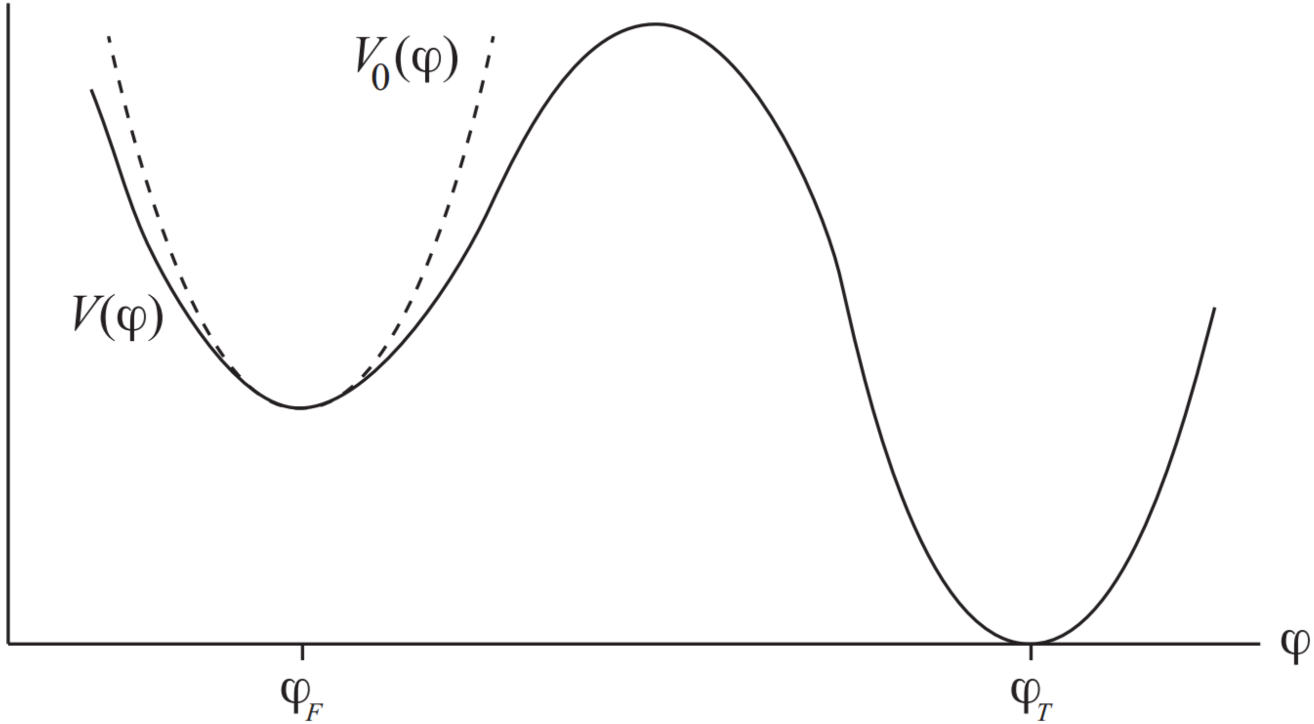
Caveat

- Of course, **need to check** whether the inflaton decoheres *during* inflation. Expect no decoherence, by comparison to massive scalar in pure dS. But working on doing this carefully.

Harder Application: Multiple Vacua

- Warning: beyond this point statements will get much less quantitative. Assign credence accordingly…

A Toy Model



- Consider a potential with two minima.
- The true ground state differs from the ground state of the perturbative approximation of the potential around the lower minimum.

True vs. Perturbative Vacua

- We could get from the perturbative to the true ground state by incorporating instanton corrections (c.f. QCD).
- But we shouldn’t think of these corrections as dynamical processes. Reality does the full nonperturbative calculation, as usual.
- Accordingly, we expect that the *true* ground state is the one that corresponds to a semiclassical geometry.

Semiclassical Geometries

- The perturbative minima are eigenstates of *field value*. But the semiclassical geometries should be eigenstates of *energy density*. This is what corresponds to a value of H, Λ, T, …
- Again, the difference between field value eigenstates and energy density eigenstates is nonperturbative…but vitally important, since we care about long time scales.

Generalizing Cosmic No-Hair

- We boldly generalize cosmic no-hair to the case of multiple vacua.
- We expect that, in QFT with an infinite-dimensional Hilbert space, “generalized cosmic no-hair” should act to take us to the *true* ground state of the entire potential.

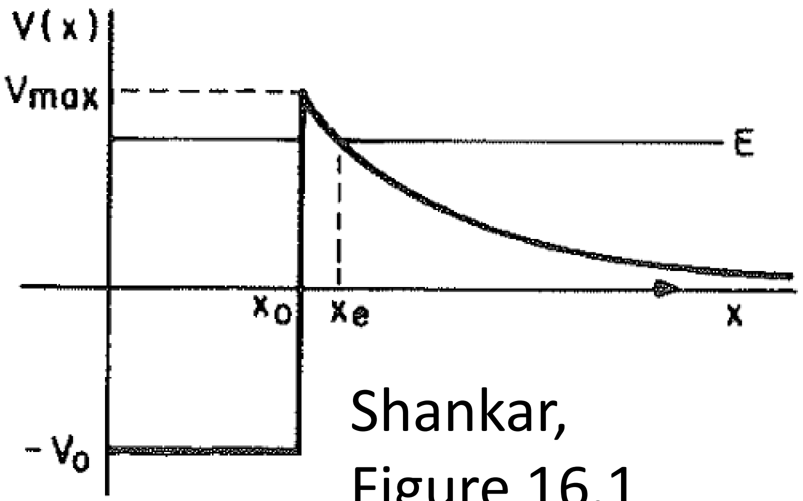
Uptunneling from the True Vacuum

- Already we can conclude that there should be no up-tunneling from the true vacuum. (Uptunneling looks like the inverse of vacuum decay, i.e. a fluctuation upward in entropy.)
- There would be uptunneling from the *perturbative* vacuum (the state of definite field value). The "rate equations" (Garriga and Vilenkin 1998,…) calculate such transitions between states of definite field value.
- Again, the math is correct, but it doesn't answer the question we want answered!

Higher Vacua

- We expect some sort of no-hair theorem should apply even for states that stay in the metastable vacua.
- Recall the WKB approximation for barrier penetration in 1d QM:

$$\frac{\psi(x_e)}{\psi(x_0)} = \exp\left(-\frac{1}{\hbar}\int_{x_0(E)}^{x_e(E)} \sqrt{2m(V(x)-E)}dx\right) \equiv e^{-\gamma/2}$$



Shankar, Figure 16.1

Tunneling Rate $R = f(E)e^{-\gamma}$

$\frac{dR}{dE} > 0$: excited modes decay faster

No-Hair for Higher Vacua

- Based on this exercise, we expect excited modes to decay faster. At late times most of the portion of the wave function that has not decayed will be very near the metastable vacuum. In particular:
- No brains
- No uptunneling to yet higher vacaua

Complementarity and Multiple Vacua

- As always when invoking no hair, we rely on an infinite-dimensional Hilbert space. This is true in standard QFT, but also even in complementarity if there is a terminal Minkowski vacuum (perhaps supersymmetric).

Landscape Eternal Inflation

- If the field starts in a metastable vacuum, it is still rewarded for staying there. So landscape inflation is still eternal in this sense.
- But we've introduced dependence on the initial conditions. The landscape is no longer "populated" starting from an arbitrary initial state.

Conclusions (1)

- Obvious but important point: quantum variables are not, in general, identical to classical probability distributions. The difference can be crucial!
- Stationary states are special—nothing happens in them, in particular no quantum fluctuations. This is true for subsystems as well, when the branching structure of the overall state confirms that they are truly stationary.

Conclusions (2)

- A causal patch in the de Sitter vacuum is one such subsystem. This is important because the cosmic no-hair theorem brings all states exponentially close to the vacuum at late times.
- There are no Boltzmann brains in such states.
- Similarly, we expect metastable de Sitter vacua to exhibit neither Boltzmann brains no uptunneling.

Conclusions (3)

- The observable consequences of inflation are unchanged, since reheating implies decoherence once modes re-enter the horizon.
- But the global structure of an inflationary universe is potentially very different.
- Slow-roll inflation does not generally lead to eternal inflation, since the inflaton does no undergo fluctuations during inflation.
- Uptunneling is absent from the inflationary landscape.